Convergence theorems for $\psi$-expansive and accretive mappings

Habtu Zegeye$^a$, Naseer Shahzad$^b$,*

$^a$Bahir Dar University, P.O.Box. 859, Bahir Dar, Ethiopia
$^b$Department of Mathematics, King Abdul Aziz University, P. O. B. 80203, Jeddah 21589, Saudi Arabia

Received 17 August 2005; accepted 8 November 2005

Abstract

Let $E$ be a real Banach space, and let $A : D(A) \subseteq E \to E$ be a Lipschitz, $\psi$-expansive and accretive mapping such that $\text{co}(D(A)) \subseteq \cap_{\lambda > 0} \mathcal{R}(I + \lambda A)$. Suppose that there exists $x_0 \in D(A)$, where one of the following holds: (i) There exists $R > 0$ such that $\psi(R) > 2\|A(x_0)\|$; or (ii) There exists a bounded neighborhood $U$ of $x_0$ such that $t(x - x_0) \notin Ax$ for $x \in \partial U \cap D(A)$ and $t < 0$. An iterative sequence $\{x_n\}$ is constructed to converge strongly to a zero of $A$. Related results deal with the strong convergence of this iteration process to fixed points of $\psi$-expansive and pseudocontractive mappings in real Banach spaces. The convergence results established in this paper are new for this more general class of $\psi$-expansive and accretive or pseudocontractive mappings.

MSC: 47H04; 47H06; 47H30; 47J05; 47J25

Keywords: Accretive mapping; $\psi$-expansive mapping; Pseudocontractive mapping; Banach space

1. Introduction

Let $E$ be a real normed linear space with dual $E^*$. We denote by $J$ the normalized duality mapping from $E$ to $2^{E^*}$ defined by

$$Jx = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\},$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. It is well known that if $E^*$ is strictly convex then $J$ is single-valued.

* Corresponding author.
E-mail addresses: habtuzh@yahoo.com (H. Zegeye), nshahzad@kau.edu.sa (N. Shahzad).